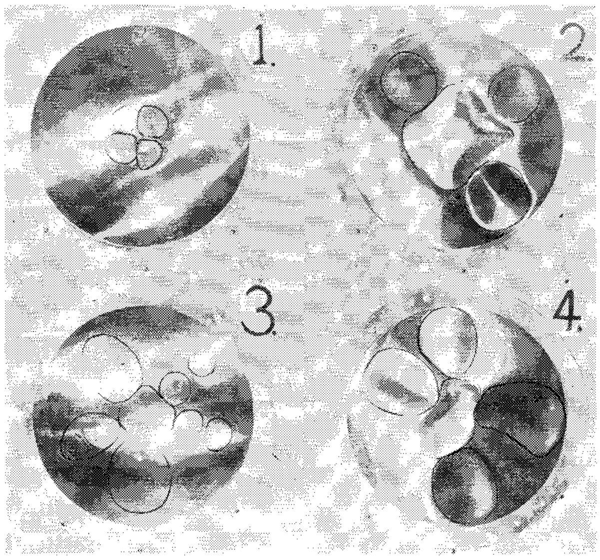

Fig. 3 Actual deformation patterns of four caps tested; thickness and drop height as follows: 1) t = 0.040 in., h = 24 in.; 2) t = 0.040 in., h = 60 in.; 3) t = 0.025 in., h = 9 in.; and 4) t = 0.025 in., h = 15 in.

mate diameter of each dimple of the permanent deformation pattern were measured. The depth was measured using the 24 in. radius template and a feeler gage or a small scale (if the depth of the dimples was greater than 0.100 in.).

Results

The results of 23 caps tested are presented in Table 1. Each cap was dropped only once. In order to aid in the interpretation of the data of Table 1, an example is given in Fig. 2. A photograph of the permanent deformations of four of the caps tested is shown in Fig. 3.

Figure 4 presents a dimensionless plot of the information given in Table 1. The results seem to represent three different types of behavior. The two points shown as x are for very thin shells for which any buckling displacement would be predominately elastic and thus would involve relatively little permanent deformation. The majority of the points, for shells of 0.025-, 0.032-, and 0.040-in. thickness, is believed to represent a situation in which the central portion of the dimples remains elastic but is held in its displaced position by the plastic hinge which has formed around each dimple. The third family, shown as $*$, is composed of the relatively thick shells (0.063 in.) for which it is believed that the central portion of the dimples as well as the surrounding hinge have been deformed plastically in bending. These arguments appear reasonable based on an elementary bending analysis which indicates that snap-through of a 24-in.-radius shell of 3003-0 aluminum would lead to inelastic bending for shells of approximately 0.040-in., and greater, thickness.

Discussion

Since axially symmetric impact of spherical caps might reasonably be expected to lead to symmetric loading and symmetric deformations, it was desired to check several possible causes for the asymmetric results obtained.

First, an investigation was made to determine the deviation of the cap axis from the line of vertical impact. As previously mentioned this was found to be less than $\pm\frac{1}{3}$ deg. A check was also made to see if the deformation patterns were influenced by the anisotropy of the cap material due to rolling. Several caps were tested with the sheet rolling direction oriented at 0, 45, and 90 deg (in the plane of the cap chord) with respect to the holding frame. This rotation produced no noticeable change in the deformation patterns. A similar check was made by rotating the cap rolling direction 0, 45, and 90 deg (in the plane of the cap chord) with respect to the

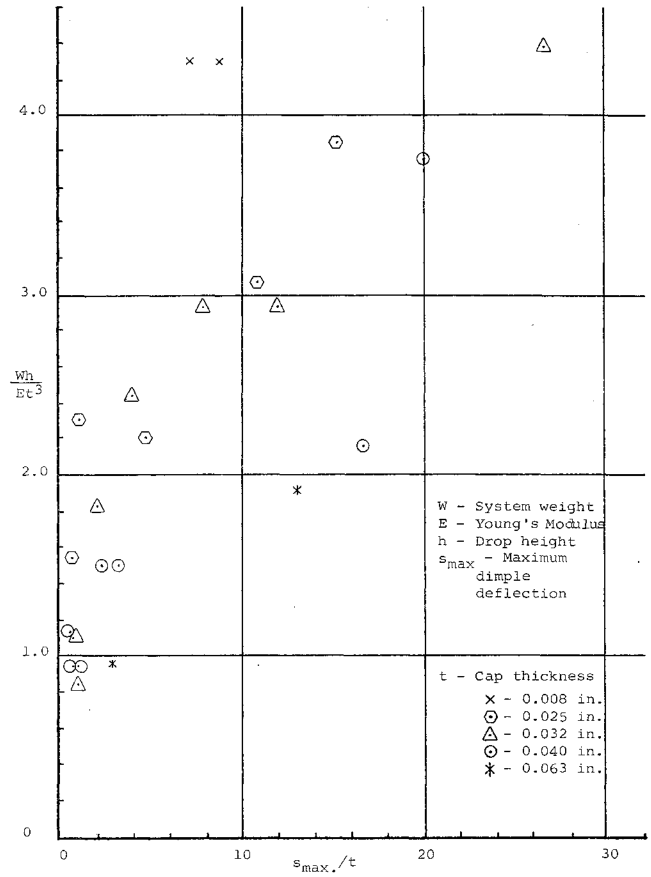

Fig. 4 Relative impact intensity vs cap maximum permanent deflection.

clamping rings. Again, no significant change was noticed in the deformation patterns.

Based on the preceding investigations, it is concluded that the results described in this note are examples of axially symmetric loading leading to asymmetric buckling response of spherical caps and that this behavior is characteristic of the system and not due to small deviations from conditions of symmetry.

Kernel Function for Nonplanar Oscillating Surfaces in a Subsonic Flow

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The relation between the velocity normal to an oscillating surface, $w = \bar{w}e^{i\omega t}$, and the pressure difference $\Delta p = \overline{\Delta p}e^{i\omega t}$ across the surface can be written as follows

$$\frac{\bar{w}}{U} = \frac{1}{4\pi\rho U^2} \iint \overline{\Delta p} K\ (x - \xi, s, \sigma, \omega, M)\ d\sigma d\xi$$

where x is the streamwise and s the tangential coordinate (see Fig. 1), ω the frequency, M the Mach number, and ξ and σ the dummy variables corresponding to x and s, respectively.

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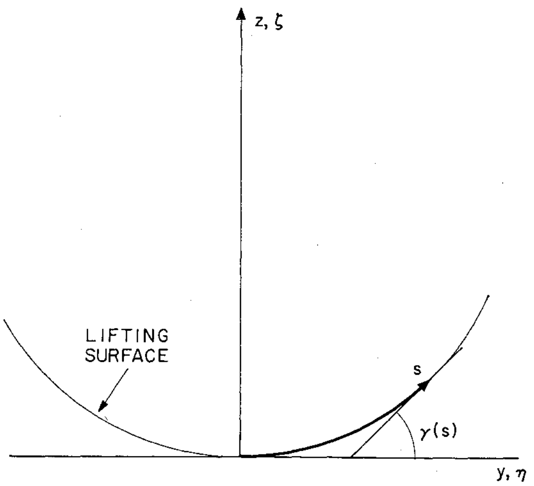

Fig. 1 Coordinate system.

The symbol ⨎ indicates that the Mangler principal value should be taken. Rodemich[1] has shown that the kernel function can be cast in the following form:

$$K = e^{-(i\omega x_0/U)}\,(K_1T_1 + K_2T_2)/r_1^2$$

where

$$K_1 = r_1\frac{\partial I_0}{\partial r_1} \qquad K_2 = r_1^2\frac{\partial}{\partial r_1}\left(\frac{1}{r_1}\frac{\partial I_0}{\partial r_1}\right)$$

$$T_1 = \cos[\gamma(s) - \gamma(\sigma)]$$

$$T_2 = \left\{\frac{z_0}{r_1}\cos[\gamma(s)] - \frac{y_0}{r_1}\sin[\gamma(s)]\right\} \times \left\{\frac{z_0}{r_1}\cos[\gamma(\sigma)] - \frac{y_0}{r_1}\sin[\gamma(\sigma)]\right\}$$

$$r_1 = (y_0^2 + z_0^2)^{1/2}$$

$$x_0 = x - \xi \qquad y_0 = y - \eta \qquad z_0 = z - \zeta$$

$$I_0 = \int_{u_1}^{\infty}\frac{e^{-ik_1u}}{(1+u^2)^{1/2}}\,du \qquad u_1 = \frac{(MR - x_0)}{\beta^2 r_1}$$

$$k_1 = \frac{\omega r_1}{U} \qquad \beta = (1 - M^2)^{1/2} \qquad R = (x_0^2 + \beta^2 r_1^2)^{1/2}$$

By setting $t = ur_1$ in the expression of the integral I_0 one finds after some simple manipulations

$$K_1 = I_1 + \frac{Mr_1}{R}\,\frac{e^{-ik_1u_1}}{(1+u_1^2)^{1/2}}$$

$$K_2 = -3I_2 - \frac{ik_1M^2r_1^2}{R^2}\,\frac{e^{-ik_1u_1}}{(1+u_1^2)^{1/2}} - \frac{Mr_1}{R}\left[(1+u_1^2)\frac{\beta^2r_1^2}{R^2} + 2 + \frac{Mr_1u_1}{R}\right]\frac{e^{-ik_1u_1}}{(1+u_1^2)^{3/2}}$$

where

$$I_1 = \int_{u_1}^{\infty}\frac{e^{-ik_1u}}{(1+u^2)^{3/2}}\,du = \int_{v_1}^{1}\exp\left[\frac{-ik_1v}{(1-v^2)^{1/2}}\right]dv$$

$$I_2 = \int_{u_1}^{\infty}\frac{e^{-ik_1u}}{(1+u^2)^{5/2}}\,du = \int_{v_1}^{1}(1-v^2)\exp\left[\frac{-ik_1v}{(1-v^2)^{1/2}}\right]dv$$

and

$$v_1 = \frac{Mr_1^2 - x_0R}{x_0^2 + r_1^2}$$

The integrals I_1 and I_2 cannot be expressed in terms of known functions but their numerical evaluation should pose no difficult problems. The expression for the planar case involves K_1 alone, and, with the first form of I_1, turns out to be identical to the one proposed by Woodcock.[2] The present expression for the nonplanar case is considerably simpler than anything previously given (e.g. Ref. 3), and in fact less complicated than the original expression[4] for the planar case.

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New MZI-Technique for Shock Tube Measurements

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It frequently is difficult to establish the continuity of interference fringes attained by a Mach-Zehnder interferometer (MZI) across a strong discontinuity, as in shock tube or shock tunnel flow.[1] A method to overcome this difficulty is known as offset interferometry.[2] By this method the beam is partially split in such a way that a reference beam traverses the interferometer at a different angle. Thus, two different interferograms are taken simultaneously, and they allow a fringe shift to follow which is discontinuous in one of them. This technique usually involves either two additional mirrors or prisms which have to be properly adjusted, or a real or virtual offset light source.[1]

A new and inexpensive method has been developed for detecting the fringe shift across a discontinuity, which consists

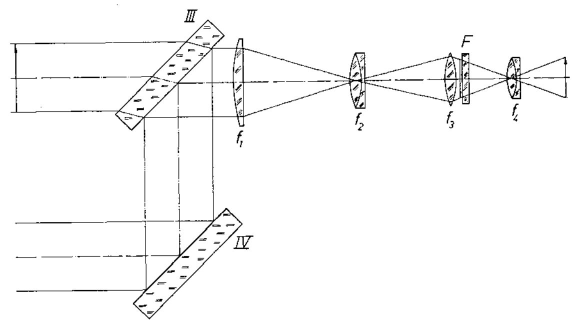

Fig. 1 Part of MZI photographing system with special filter. The lenses used here were f_1 = 127 cm, f_2 = 42 cm, f_3 = 16 cm, f_4 = 13.5 cm.

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